Maths: Year 11H- Topic 4 Overview

Geometry: Vectors

Students will have met vectors to describe translations during Key Stage 3. This will be revisited and used as the basis for looking more formally at vectors, discovering the meaning of $-\boldsymbol{a}$ compared to $\boldsymbol{a}$ to make sense of operations such as addition, subtraction, subtraction and multiplication of vectors. This will connect to exploring "journeys" within shapes linking the notation $\overrightarrow{AB}$ with $\boldsymbol{b}-\boldsymbol{a}$ etc. Higher tier students will then use this understanding as the basis for developing geometric proof, making links to their knowledge of properties of shape and parallel lines.

Specialist key terms for this unit:

Column vector, Direction, Scalar, Size, Magnitude, Arrow, Parallel, Multiplier, Equal, Opposite, Addition, Resultant, Subtraction, Multiplying, Vector journey, Express, Parallel, Negative, Equal, Opposite, Multiple, Multiplier, Fractional, Common point, Collinear, Same line, Show, Justify, Prove

Key questions:

What do the numbers in the column vector represent?
How do you know which direction they represent?
What does the arrow show?
What is the significance of the order of the letters when writing $\overrightarrow{AB}$? What does the arrow tell us?
What's the same and what's different about parallel vectors?
How do we know if one vector is a multiple of another?
How do we know if the multiplier is negative?
What's the relationship between $\boldsymbol{b}$ and $-\boldsymbol{b}$?
Which quadrilaterals will have pairs of parallel vectors?
How many pairs of parallel vectors will they have?
Can a square be described by four equal vectors? Why not? What is the same and what's different about the four vectors that describe a square?
What about the vectors that describe a rectangle?
What do you know? What can you find out?
What do the words 'show, justify and prove' mean?

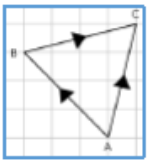
Practise makes perfect

https://www.mathsgenie.co.uk/column-vectors.html

https://www.mathsgenie.co.uk/vectors.html

Exemplar Questions

$\overrightarrow{AB} = \begin{pmatrix}-3\\3\end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix}4\\1\end{pmatrix}$



Complete: $\overrightarrow{AB} + \overrightarrow{BC} = \begin{pmatrix}-3\\3\end{pmatrix} + \begin{pmatrix}4\\1\end{pmatrix} = \begin{pmatrix}1\\?\end{pmatrix}$

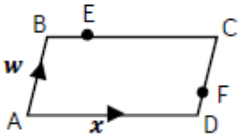
Compare this to the column vector representing $\overrightarrow{AC}$

Eva and Alex are thinking about how they can find $\overrightarrow{AC}$ using $\overrightarrow{AB}$ and $\overrightarrow{BC}$. Who is right? Why?

Eva $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{CA}$ Alex $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

ABCD is a parallelogram.

BE : EC = 1 : 3 and CF = $\frac{3}{4}$CD.



Express in terms of $\boldsymbol{x}$ and $\boldsymbol{w}$ $\overrightarrow{BD}$ $\overrightarrow{BE}$ $\overrightarrow{AE}$ $\overrightarrow{AF}$

Show that $\overrightarrow{BD}$ is parallel to $\overrightarrow{EF}$

Assessment / Topic Reviews:

1. Understand and represent vectors
2. Use and read vector notation
3. Draw and understand vectors multiplied by a scalar
4. Draw and understand addition of vectors
5. Draw and understand addition and subtraction of vectors
6. Explore vector journeys in shapes – H
7. Explore quadrilaterals using vectors – H
8. Understand parallel vectors - H
9. Explore collinear points using vectors – H
10. Use vectors to construct geometric arguments and proofs - H

Suggested enrichment

Vector walk

Suppose that I am given a large supply of basic vectors $b_1 = \begin{pmatrix}2\\1\end{pmatrix}$ and $b_2 = \begin{pmatrix}0\\1\end{pmatrix}$.

Starting at the origin, I take a 2-dimensional 'vector walk' where each step is either a b_1 vector or a b_2 vector, either forwards or backwards.

Investigate the possible coordinates for the end destinations of my walk.

Can you find any other pairs of basic vectors which yield exactly the same set of destinations?

Can you find any pairs of basic vectors which yield none of these destinations?

Can you find any pairs of basic vectors which allow you to visit all integer coordinates?

Maths: Year 11H- Topic 5 Overview

Functions

This unit develops students' understanding of functions and their applications in algebra and graphs. Students will use function machines, substitute into expressions and formulae, and work with function notation. They will also explore composite and inverse functions (Higher Tier), investigate quadratic graphs and inequalities, and learn how to use trigonometric functions to solve mathematical problems.

Specialist key terms for this unit:

Input, Output, Function, Operation, Inverse, Variable, Evaluate, Substitute, Expression, Formulae, Solve, Quadratic, Graph, Intercept, Turning Point, Roots, Ratio, Sine, Cosine, Tangent, Opposite, Adjacent, Hypotenuse, Function, Composite, Inverse, Rearrange, Inequality, Represent, Set, Solution

Key questions:

If the input is ….., what is the output? How do you know?

If the output is…., what is the input? How do you know?

How can you check your answer?

What does it mean to substitute a value?

What is the difference between the expressions $x - 7$ *and* $7 - x$?

Choose values for x to show that $5x^2 \neq (5x)^2$

What's different between f(x) and f(2)?

What's different between f(x) and f(a)?

What's different between f(x) and g(a)?

What does fg(x) mean?

What gf(x) mean?

What does ff(x) mean?

Calculate gf(5) and fg (5). What's the same? What's different?

Practise makes perfect

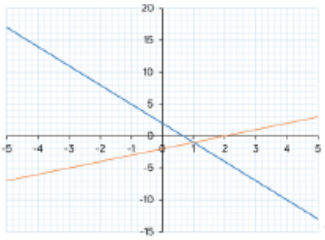
https://www.mathsgenie.co.uk/functions.html

Exemplar Questions

Given that $f(x) = 7x + 11$ and $g(x) = 10 - x$, work out

- $f(2)$
- $g(2)$
- $fg(2)$
- $gf(2)$
- $ff(2)$
- $gg(2)$

Two functions, $f(x)$ and $g(x)$ are shown on the graph.



- Solve $f(x) = g(x)$
- Find an expression for $f^{-1}(x)$
- Find an expression for $g^{-1}(x)$
- Solve $f^{-1}(x) = g^{-1}(x)$

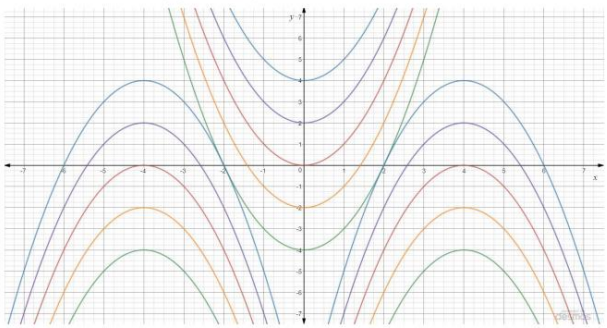
Assessment / Topic Reviews:

1. Use function machines
2. Substitution into expressions and formulae
3. Use function notation
4. Work with composite functions – H
5. Work with inverse functions – H
6. Graphs of quadratic functions
7. Solve quadratic inequalities – H
8. Understand and use trigonometric functions

Suggested enrichment activities:

nrich **Parabolic Patterns**

The graphs of fifteen parabolas are sketched on the axes below.



One of them has the equation $y = x^2$, and another is $y = -(x-4)^2$.
Can you find the equations of the other parabolas?

Maths: Year 11H- Topic 6 Overview

Graphs: Gradients and lines

This topic builds on earlier study of straight line graphs in year 9 and 10. Students will plot straight lines from a given equation, and find and interpret the equation of a straight line from a variety of situations and given information. Higher tier also will study the equations of perpendicular lines.

Key words

Angle, Adjacent, Vertically opposite, Point, Full turn, Straight line, Parallel, Corresponding, Alternate, Bearing, Co-interior, Polygon, Regular, Interior, Exterior, Equilateral, Isosceles, Right-angle, Trapezium, Vector, Column, Horizontal, Vertical, Position, Triangle, Opposite, Adjacent, Hypotenuse, Ratio, Inverse, Circle, Segment, Circumference, Centre, Right-angle, Cyclic quadrilateral, Radius, Chord, Bisects, Tangent, Length, Equal

Key questions

What is the equation of the x-axis?

What is the equation of the y-axis?

What is the minimum number of points needed to plot a straight line graph?

In y = mx + c, what do m and c represent?

What does it mean when two lines have the same gradient?

What does it mean when two lines have the same y-intercept?

What is the scale on each axis? How does the scale affect the gradient?

How many solutions do a pair of simultaneous equations have?

When two lines are perpendicular, why must one gradient be positive and one be negative?

Practise makes perfect

https://www.mathsgenie.co.uk/lineargraphs.html

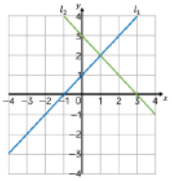
https://www.mathsgenie.co.uk/gradient-of-a-line.html

https://www.mathsgenie.co.uk/equation-of-a-line.html

https://www.mathsgenie.co.uk/parallel-and-perpendicular-lines.html

Exemplar Questions

Two lines, l_1 and l_2 are shown on the graph.



- What is the equation of l_1?
- What is the equation of l_2?
- What are the coordinates of the point of intersection?

Assessment / Topic Reviews

1. Parallel lines to the axis x=a and y=a
2. Plot straight line graphs
3. Interpret y=mx + c
4. Find the equation from a straight line from a graph
5. Find de equation of a straight-line graph given one point and a gradient
6. Find de equation of a straight-line graph given two points
7. Determine whether a point is on a line
8. Solve linear simultaneous equations graphically
9. Recognise when straight lines are perpendicular (H)
10. Find the equaition of perpendicular lines (H)

Suggested enrichment activities

Designing a Skate Park

A new skate park is being designed, and the ramps are modelled using straight-line graphs.

1. The first ramp is represented by the line $y = 3x + 2$. Plot the line.
2. Check whether the point (2, 8) lies on the ramp.
3. A second ramp crosses the first at a right angle and passes through (0, 5). Find its equation and draw it.
4. Find the point where the two ramps meet.

Challenge: Explain how the gradients show that the ramps are perpendicular and why this might be useful in the skate park design.

Maths: Year 11H- Topic 7 Overview

Graphs: Non-linear graphs

In this topic, students develop their knowledge of non-linear graphs, looking at quadratic, cubic and reciprocal graphs so they recognise the different shapes. They find the roots of quadratics graphically. Higher tier also looks at simple exponential graphs and the equation of a circle. Also, higher tier will extend their understanding of gradient to include instantaneous rates of change considering the gradient of a curve at a point.

Key words

Quadratic, Parabola, Curve, Substitute, Equation, Vertical, Horizontal, Estimate, Cube, Cubic, Asymptote, Infinity, Reciprocal, Tends towards, Gradient, y-intercept, Coordinate, Roots, Solution, Meets, Exponential, Growth, Decay, Rapd, Tends, Radius, Diameter, Pythagoras' theorem, Origin, Simplify, Exponential, Growth, Decay, Rapid, Tends, Tangent, Curve, Equidistant

Key questions

Why is $(-3)^2$ the same as 3^2?

How could I tell if one of my coordinates was incorrect, or if I had plotted it incorrectly?

Why do we join the points with a smooth curve when plotting a quadratic?

What do we mean by infinity?

What are the key features of this graph?

If you are not sure which equation matches a graph, what could you do to find out more information?

How can we locate roots from a graph?

How is Pythagoras' Theorem connected to the equation of a circle?

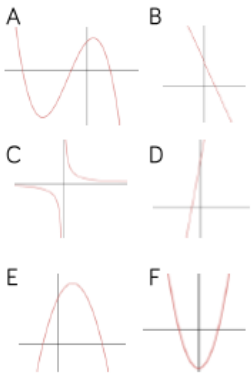
What are the steps to draw a tangent to a curve at a given point?

Practise makes perfect

https://www.mathsgenie.co.uk/quadratic-graphs.html

https://www.mathsgenie.co.uk/cubic-reciprocal.html

Match each graph with its equation.



Equation	Letter	Type of Graph
$y = 10x + 10$		
$y = \frac{1}{x}$		
$y = x^2 - 10$		
$y = x^3$		
$y = -x^2 + 2x + 3$		
$y = -2x + 4$		
$y = -x^3 - 2x^2 + x + 1$		

One equation in the table doesn't have a match.
Sketch a graph to match this equation.

Assessment / Topic Reviews

1. Plot and read from quadratic, cubic and reciprocal graphs
2. Recognise graphs shapes
3. Identify and interpret roots and intercepts of quadratics
4. Understand and use exponential graphs (H)
5. Find and use the equation of a circle (H)
6. Equation of a tangent to a any curve (H)

Suggested enrichment activities

Choosing the Right Graph

A company wants to model different situations using graphs.

Tasks:

1. Match quadratic, cubic and reciprocal graphs to these situations:
   - A ball being thrown into the air
   - Profit increasing, then decreasing, then increasing again
   - Journey time for a fixed distance
2. Explain your choices.

⭐ Challenge: Draw a quick sketch of each graph and label one key feature.